Geometry of attosecond laser pulses and photon-photon scattering at high energies

Kirill Tuchin [a,b]
[a] Department of Physics and Astronomy,
Iowa State University, Ames, IA 50011
[b] RIKEN BNL Research Center, Upton, NY 11973-5000

(Dated: September 11, 2009)

We derive the total cross section for scattering of a photon on an ultra-short laser pulse at high energies. We take into account all multi-photon interactions. We argue that the nonlinear effects due to these interactions become important at very high intensities of the laser pulse. We demonstrate however, that these intensities are significantly lower than the Schwinger critical value.

arXiv:0909.2195v1 [hep-ph] 11 Sep 2009

Geometry of a laser pulse ℓ plays a prominent role in its interactions with elementary particles [1, 2]. In this letter we consider interactions of photons with ultra-short laser pulses at high energies. Laser pulse is a coherent state of large number of photons N. If width $2d$ and length L of the laser pulse can be neglected, then the total $\gamma\ell$ cross section reads [3, 4]

$$\sigma_{\rm tot}^{\gamma\ell}(s)=\frac{\alpha^4 N}{m^2}\frac{1}{36\pi}[175\zeta(3)-38]\,. \tag{1}$$

This is just N times the total $\gamma\gamma$ cross section, which at high energies receives the leading contribution from the process $\gamma\gamma\to e^-e^+e^-e^+$ mediated by the Weizsäcker-Williams photon exchanged in the t-channel between the two e^+e^- electric dipoles. At high intensities I of the laser pulse, i.e. at large N, close to the Schwinger critical value, the non-perturbative effects as well as non-linear perturbative ones are expected to produce large corrections to Eq. (1). In this letter we demonstrate that it is in principle possible to prepare a laser pulse that would interact non-linerly with energetic photons at intensities smaller than the critical.

We begin with an observation that the laser pulse width $2d$ is actually large compared to the effective radius of the electro-magnetic interactions. Indeed, the maximal impact parameter between the two e^-e^+ pairs is $b'_{\max}=\frac{\sqrt{s}}{4m^2}$, which is the consequence of existence of the minimal longitudinal momentum transfer [6] and the fact that the t-channel photon is almost real $l^2\approx-\mathbf{l}^2$ (we use bold face for transverse – w.r.t. the collision axes – two-vectors). Here s and t are the usual Mandelstam variables. For realistic values of s and d it holds that $b'_{\max}\ll d$. For example, consider collision of $\omega=100$ GeV photon with $\Omega=100$ eV laser pulse. The high energy approximation holds since $\sqrt{s}=6.3\gg 1$ MeV. The pulse width can be estimated as $d\sim\lambda=2\pi/\Omega=1.3$ nm; hence $b'_{\max}/d=6\cdot10^{-3}\ll 1$. We will often refer to this example.

To take the finite width of the laser pulse into account it is convenient to Fourier transform the amplitude of $\gamma\ell$ scattering into the transverse configuration space. Then, the wave-function $\tilde\Phi(\mathbf{r})$ of the energetic photon, describing splitting of a photon into e^-e^+ dipole, factorizes from the imaginary part of elastic scattering amplitude $i\Gamma^{d\ell}$, where d stands for the electric dipole e^-e^+ produced by the energetic photon. Let $\mathbf{b}'$ be the impact parameter between the dipole d and a photon γ_ℓ in the pulse; $\mathbf{B}$ – between the dipole d and the pulse symmetry axes and $\mathbf{b}$ – between the photon γ_ℓ and the symmetry axes; clearly, $\mathbf{b}'=\mathbf{B}-\mathbf{b}$. The total cross section reads [7, 8]

$$\sigma_{\rm tot}^{\gamma\ell}(s)=\frac{1}{2!}\int d^2B\int\frac{d^2r}{2\pi}\,\tilde\Phi(\mathbf{r})\,2\,\langle \mathrm{Im}\,[i\Gamma^{d\ell}(\mathbf{r},\mathbf{B},s)]\rangle\,, \tag{2}$$

where [9]

$$\tilde\Phi(\mathbf{r})=\frac{\alpha m^2}{\pi}\left(\frac{2}{3}K_1^2(mr)+K_0^2(mr)\right). \tag{3}$$

The longitudinal extent L of the laser pulse is also an important parameter. It is determined by the pulse duration τ. For an attosecond pulse $\tau\sim10^{-18}$ sec which translates into $L\sim\tau c=0.3$ nm. For highly energetic photons L may turn out to be shorter than the coherence length l_c of the energetic photon*, which is given by the inverse of the minimal longitudinal momentum transfer. In the Lab frame [6]

$$l_c=(\hbar\omega/2mc^2)\lambda\,, \tag{4}$$

where $\lambda=3.8\cdot10^{-4}$ nm is the Compton wavelength of electron. Coherence effects due to multiple scattering of an energetic photon on photons of the laser pulse are important when $l_c\gg L$ which implies a condition

$$\frac{\hbar\omega}{2mc^2}\gg\frac{L}{\lambda}\,. \tag{5}$$

In the following we assume that (4) is satisfied. In particular, for a sample parameter set chosen above $l_c/L\simeq 200$.

Since all N photons in the pulse are in a coherent state we can write using the Glauber approach [10]

$$\langle \mathrm{Im}\,[i\Gamma^{d\ell}(\mathbf{r},\mathbf{B},s)]\rangle=\mathrm{Im}\,(1-e^{-N\langle i\Gamma^{d\gamma_\ell}(\mathbf{r},\mathbf{B},s)\rangle})\,, \tag{6}$$

* This is not to be confused with the laser coherence length.

where $i\Gamma^{d\gamma_\ell}$ is elastic scattering amplitude of the dipole d on a photon γ_ℓ. Averaging in (2) and (6) is performed over the entire volume of the pulse. At high energies, the interaction is approximately instantaneous (since the mediating photons are almost real) implying that the dipoles in the pulse do not recoil. One then describes the number distribution of the dipoles in the pulse by a function $\rho(\mathbf{b}, z)$, where z is the longitudinal position of a dipole. It is normalized such that

$$\int d^2b\, dz\, \rho(\mathbf{b}, z) = N\,. \tag{7}$$

The average of the amplitude over the dipole position in the pulse reads

$$\langle i\Gamma^{d\gamma_\ell}(\mathbf{r}, \mathbf{B}, s)\rangle = \frac{1}{N}\int d^2b \int_{-L/2}^{L/2} dz\, \rho(b)\, i\Gamma^{d\gamma_\ell}(\mathbf{r}, \mathbf{B}-\mathbf{b}, s). \tag{8}$$

We estimated above that $b'_{\max} \ll d$. Since $b \sim d$ it implies that $b' \ll B \approx b$. Neglecting for notational simplicity z-dependence of ρ we write

$$\langle i\Gamma^{d\gamma_\ell}(\mathbf{r}, \mathbf{B}, s)\rangle \approx \frac{L}{N}\,\rho(B)\int d^2b' i\Gamma^{d\gamma_\ell}(\mathbf{r}, \mathbf{b}', s)\,. \tag{9}$$

To the leading order in α the dipole-photon elastic scattering amplitude is depicted in Fig. 1. We can write analogously to (2)

$$\mathrm{Im}\,[i\Gamma^{d\gamma_\ell}(\mathbf{r}, \mathbf{b}', s)] = \int \frac{d^2r'}{2\pi}\tilde{\Phi}(r')\,\mathrm{Im}\,[i\Gamma^{dd}(\mathbf{r}, \mathbf{r}', \mathbf{b}', s)]\,, \tag{10}$$

where $i\Gamma^{dd}$ is the elastic dipole–dipole scattering amplitude.

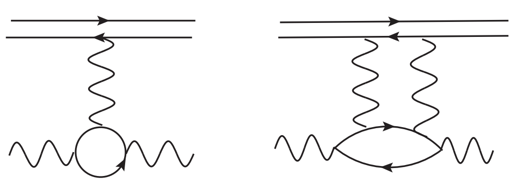

FIG. 1: Leading order contributions to the dipole-photon elastic cross section. The upper dipole is associated with the energetic photon. The horizontal wavy line is a photon in the laser pulse.

Diagram Fig. 1(a) represents the leading contribution to the real part of the dipole-photon scattering amplitude. However, it vanishes due to the C-invariance of QED (Furry's theorem). Higher order contributions to the real part of the dipole–photon amplitude are represented by diagrams with odd number of exchanged photons. They vanish for the same reason. Therefore, the dipole-photon amplitude is purely imaginary at high energies. The leading diagram is shown in Fig. 1(b). The corresponding dipole–dipole amplitude is

$$i\Gamma^{dd} = 2i\alpha^2 \ln^2 \frac{|\mathbf{b}' + \frac{1}{2}\mathbf{r} + \frac{1}{2}\mathbf{r}'||\mathbf{b}' - \frac{1}{2}\mathbf{r} - \frac{1}{2}\mathbf{r}'|}{|\mathbf{b}' + \frac{1}{2}\mathbf{r} - \frac{1}{2}\mathbf{r}'||\mathbf{b}' - \frac{1}{2}\mathbf{r} + \frac{1}{2}\mathbf{r}'|}\,. \tag{11}$$

Using (2),(6),(9),(10),(11) we derive for the total pair-production cross section

$$\sigma_{\mathrm{tot}}^{\gamma\ell} = \frac{\alpha}{\pi}\int d^2B \int_0^\infty duu\,\tilde{\Phi}(u)\big\{1 - e^{-\kappa(u,B)}\big\}\,, \tag{12}$$

where

$$\kappa = \frac{8\alpha^3\rho(B)\,L}{m^2}u^4\int_0^1 d\xi \log\frac{e}{\xi}\big[\xi^3\tilde{\Phi}(u\xi) + \xi^{-3}\tilde{\Phi}(u\xi^{-1})\big] \tag{13}$$

with $\xi = r'/r$ and $u = mr$.

When $\kappa = N\langle i\Gamma^{d\gamma_\ell}\rangle \ll 1$ the exponent in (6) and (12) can be expanded which corresponds to the linear two-photon exchange regime. Integration over dipole sizes and impact parameters then yields (1). In the opposite limit $\kappa \gg 1$ the nonlinear effects are strong leading to saturation of the cross section at its geometric limit $\sigma_{\mathrm{tot}}^{\gamma\ell} \sim \alpha\, d^2 \ln\kappa$. This regime is characterized by weak logarithmic dependence of the cross section on intensity of the laser pulse. Given the laser pulse of wavelength λ and pulse duration τ, transition from the linear to the saturation regime can be characterized by intensity I_{mp} for which $\kappa \simeq 1$. We can estimate

$$\kappa \simeq \frac{8\alpha^3\tau\lambda\lambda^2 I}{\pi\hbar c}\,. \tag{14}$$

Therefore

$$I_{\mathrm{mp}} = \frac{\pi\hbar c}{8\alpha^3\tau\lambda\lambda^2}\,. \tag{15}$$

Let us compare I_{mp} with the critical value I_c at which the Schwinger mechanism of pair production [11, 12, 13] becomes important. $I_c = cE_c^2$ where the critical value of electric field is $E_c \sim m^2c^3/(e\hbar)$. Therefore,

$$\frac{I_{\mathrm{mp}}}{I_c} \sim \frac{1}{\alpha^2}\frac{\lambda^2}{L\lambda} \ll 1 \tag{16}$$

for realistic values of L and λ. In fact, for the sample set of parameters that we are using $I_{\mathrm{mp}} = 7\cdot 10^{-5}\, I_c$.

In conclusion, we argued that the geometry of attosecond laser pulses is such that at presently accessible high energies `(i)` the width of the pulse d is much larger than the effective radius electro-magnetic interactions, `(ii)` length of the pulse L is shorter than the coherence length. For this case we derived the total cross section (12) that includes all multi-photon interactions between the energetic photon and the laser pulse. These multi-photon interactions are important at laser intensities I_{mp} much smaller than the critical value at which the perturbative vacuum of QED becomes unstable. This result may be

useful for investigation of nonlinear effects in electrodynamics (see e.g. [14]).

This work was supported in part by the U.S. Department of Energy under Grant No. DE-FG02-87ER40371. I thank RIKEN, BNL, and the U.S. Department of Energy (Contract No. DE-AC02-98CH10886) for providing facilities essential for the completion of this work.

[1] V. N. Baier, V. M. Katkov and V. M. Strakhovenko, Phys. Lett. B **229**, 135 (1989).
[2] K. Tuchin, arXiv:0907.5189 [hep-ph].
[3] L. N. Lipatov and G. V. Frolov, Pisma Zh. Eksp. Teor. Fiz. **10**, 399 (1969); Yad. Fiz. **13**, 588 (1971).
[4] H. Cheng and T. T. Wu, Phys. Rev. D **1**, 3414 (1970).
[5] V. N. Baier, E. A. Kuraev, V. S. Fadin and V. A. Khoze, Phys. Rept. **78**, 293 (1981).
[6] V. B. Berestetsky, E. M. Lifshitz and L. P. Pitaevsky, "Quantum Electrodynamics," §93, *Oxford, Uk: Pergamon (1982) 652 P. (Course Of Theoretical Physics, 4).*
[7] A. Donnachie, H. G. Dosch and M. Rueter, Eur. Phys. J. C **13**, 141 (2000) [arXiv:hep-ph/9908413].
[8] S. Bondarenko, M. Kozlov and E. Levin, Acta Phys. Polon. B **34**, 3081 (2003) [arXiv:hep-ph/0303118].
[9] N. N. Nikolaev and B. G. Zakharov, Z. Phys. C **49**, 607 (1991).
[10] R. J. Glauber, *In *Lo, S.Y. (ed.): Geometrical pictures in hadronic collisions*, 83-182. World Scientific* (1987).
[11] F. Sauter, Z. Phys. **69**, 742 (1931).
[12] W. Heisenberg and H. Euler, Z. Phys. **98**, 714 (1936) [arXiv:physics/0605038].
[13] J. S. Schwinger, Phys. Rev. **82**, 664 (1951).
[14] M. Marklund and P. K. Shukla, Rev. Mod. Phys. **78**, 591 (2006) [arXiv:hep-ph/0602123].